# A Scheme for a High Power, Low Cost Transmitter for Deep Space Applications

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**Abstract.** Applications such as planetary radars and spacecraft communications require transmitters with extremely high effective isotropic radiated power (EIRP). Until now, this has been done by combining a high power microwave source with a large reflective antenna. However, this arrangement has a number of disadvantages. It is costly, since the steerable reflector alone is quite expensive, and for spacecraft communications the need to transmit hurts the receive performance. For planetary radars the utilization is very low since the antenna must be shared with other applications such as radio astronomy or spacecraft communications. This paper describes a potential new way of building such transmitters with lower cost, greater versatility, higher reliability, and potentially higher power. The basic idea is a phased array with a very large number of low power elements, built with mass production techniques that have been optimized for consumer markets. The antennas are built en mass on printed circuit boards, and driven by chips, built with consumer CMOS technology, that adjust the phase of each element. Assembly and maintenance should be comparatively inexpensive since the boards need only be attached to large, flat, unmoving, ground-level infrastructure. Applications to planetary radar and spacecraft communications are examined. Although we would be unlikely to use such a facility in this way, an implication for SETI (Search for Extraterrestrial Intelligence) is that high power beacons are easier to build than had been thought.

## 1. Introduction

Powerful microwave transmitters are essential for deep space research. They are a crucial component of planetary radar, used to study planets and asteroids, and they provide command and control for distant spacecraft. These applications require transmitters with extremely high EIRPs, in the range of 10s of Terawatts(TW, or $10^{12}$ watts). The conventional (and so far only) approach to generate such high EIRPs is to combine a high power microwave source with a large antenna. As examples, the planetary radar at Goldstone uses a 500 KW transmitter with a 70 meter reflector [*Freiley et al.*, 1992a], and the radar at Arecibo uses a 1 MW transmitter with a 305 meter reflector [*Castleberg and Xilouris*, 1997]. Although effective, this arrangement has some serious disadvantages. The planetary radars largely sit idle (they are used as radars only a few percent of the time), primarily because the associated antennas are also in high demand for radio astronomy and spacecraft communications. For spacecraft communications, the need to use the same antenna both for receiving and transmitting means that dichroic splitters and special filters are required, hurting the receive performance. Only one target can be illuminated at a time, in general, because of the small field of view of a large dish antenna. High winds can disable the ability to transmit by making it impossible to point the antenna with the requisite accuracy, or by requiring the antenna to be stowed to avoid physical damage. High power microwave transmitters require specialty items such as high power tubes, cables, and waveguides [*Cormier and Mizuhara*, 1992; *Freiley et al.*, 1992b]. These are long lead time items, finicky to operate, and pose a long term reliability worry. Construction may be expensive, and maintenance difficult, because the transmitter must be mounted at a focus of the antenna. This location is high off the ground, constrained in volume, and changes orientation as the dish moves.

This paper describes a method for building a cheaper transmit-only facility that is also more flexible, more reliable, and potentially higher power. The basic idea is to construct the transmitter with a very large number of small, cheap, low power elements, to take advantage of the fact that with correct phasing the EIRP grows as the square of the number of elements. The basic unit of construction is a *tile*, consisting of a printed circuit (PC) board with many printed antennas, and a single integrated circuit (IC). The IC (also called a *chip*) drives all antennas on a board, controlling each of them individually in phase. Each tile should be relatively inexpensive since the components can be mass produced in factories already optimized for large volumes of consumer goods. The tiles are mounted upon a large, flat, ground-level surface (think of the huge expanse of floor tile at a large shopping mall). Assembly and maintainability should be good since there are no moving parts and everything is at ground level.

As a numerical example, a transmitter could use $5 \cdot 10^7$ elements of 10 mW each. This gives a total EIRP of 33 TW (given various assumptions detailed later), about twice that of Arecibo. At X-band, about 100 antennas fit on a tile. This array requires 500K tiles; if each tile can be built for $20, as seems likely, then the total cost of the array elements will be $10M. The supporting structure, array control, power supplies, assembly labor, and so on will add to the cost. However, the supporting structure only needs to be stable and not accurate, and all assembly occurs at ground level, accessible by truck and foot. Therefore the assembly should be cheap (perhaps $5-6 million with reasonable assumptions) compared to the assembly of a conventional radio telescope, which requires skilled and specialized labor. In comparison, the last large radio telescope built in the US

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was the fully steerable 100 meter paraboloid at Green Bank, which cost about $100 million.

This arrangement has advantages in addition to cost. Reliability should be high, since with proper design no individual element is crucial, and elements can be replaced while the transmitter is running. Unlike a dish, the transmitter could be used in any wind conditions. (The effects of rain are the same, and the effects of snow may be worse since the antenna cannot be tipped to dislodge the snow). Also unlike a dish, multiple beams in very different directions can be generated simultaneously, though the total available power must be shared. Furthermore, the design is low risk, since a much smaller prototype with identical parts can be characterized before large quantities are ordered. The array can be expanded at any time, even after initial construction is complete, if more power is needed. For spacecraft commanding, a separate transmit only facility removes the need for each receiving antenna to simultaneously transmit, which allows a helpful gain in receive efficiency. For planetary radar, the efficiency of operations would be considerably improved since a separate transmitter means the entire allocation of radio telescope time (the hardest asset to acquire) could be devoted to receiving.

## 2. Previous work

Conventional planetary radar transmitters are described in *Freiley et al.* [1992a] and *Castleberg and Xilouris* [1997]. A recent overview of asteroid research by radar is available [*Ostro et al.*, 2002], along with a much more extensive bibliography [*Ostro*, 2003]. Spacecraft uplink requirements are described in a JPL document[*JPL*, 2003].

There has been an enormous amount of work in the field of phased array radars in general. A search of the IEEE data base alone shows more than 500 papers on this topic. *Brookner* [2002] presents an overview. In general this work considers much smaller numbers (at most tens of thousands, instead of tens of millions) of much higher powered elements (watts and up, compared to milliwatts in the current work) [*Kopp et al.*, 2002].

Phased array transmitters for planetary applications were considered in the work of *Dickinson et al.* [1999], but the approach there used a small number (2) of large transmitters and antennas.

Small signal work integrating many phased elements on a chip (though for receive, not transmit) is covered by *Hashemi et al.* [2004].

For spacecraft, dedicated transmit-only facilities have been proposed [*Cornish*, 2001], but are seldom useful since it is unusual to have several different spacecraft in the same (very small) field of view of a large parabolic antenna (one exception is Mars, which is the case for which the dedicated uplink facility was proposed.)

## 3. Overall Design

The basic design is a very large number of small transmitters, individually controllable in phase. The basic unit of construction is a tile, consisting of a PC board mounted a few mm above a flat metal plate. The PC board has an array of antennas printed on it, all driven by a single chip that contains a programmable phase shifter and low power RF amplifier for each antenna. A central computer sends commands to each tile's chip to control the phases of the elements and modulate the beam(s). For an X-band transmitter, a tile would be roughly 25 cm on a side and contain about 100 antennas.

Physically, the tiles are mounted on a large flat surface such as a concrete slab. The main array is divided into quarters, so a portion of the array can be shut down to allow servicing without taking the array out of service (though the maximum available power would be reduced, of course). Access roads and paths allow easy maintance. A golf cart size vehicle can be driven within a few meters of any point in the array, and a worker on foot, and at ground level, can then service or replace any element.

Each chip receives its master phase reference from a central source via a network of cables, most likely fiber optics. Phase stability is obtained by a combination of physical stability of the distribution network and periodic calibration.

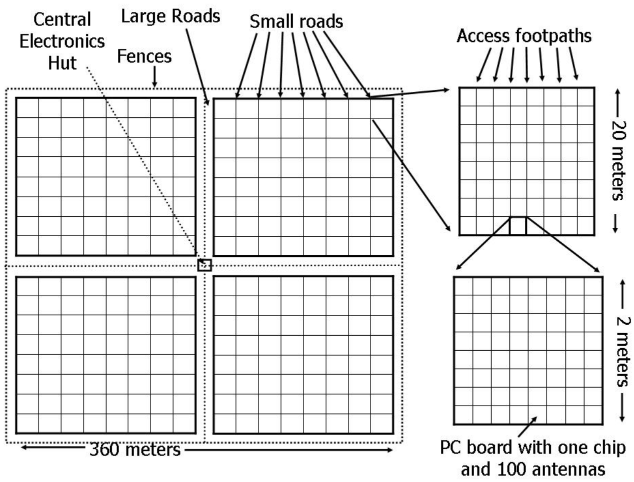


**Figure 1.** Overview of the physical design of the array.

### 3.1. Design of the board

A possible board design is shown in Fig. 2. Printed arrays of antennas such as proposed here are well known, and have been extensively studied, measured, and used in applications (see *Ashkenazy et al.* [1986] or *Au et al.* [1996], for example). Fig. 2 shows dipoles for simplicity. In practice, both spacecraft commanding and planetary radar use circularly polarized beams. There are many designs for printed antennas with circular polarized radiation [*Li et al.*, 2003; *Nesic et al.*, 1998; *Ravipati and Shafai*, 1999; *Luk and Ka-Yan*, 2003; *Lee et al.*, 1996] that have appropriate characteristics. They offer roughly 6 dB gain on axis and about a 90 degree wide radiation pattern, down about 3 dB at that point. Since a 3 dB gain was used for the EIRP calculation, an array comprised of the above elements could generated the proposed EIRPs anywhere within 45 degrees of the zenith.

Since the elements of an array antenna are closely packed, they interact strongly. The author has been unable to find any studies of closely packed circularly polarized antennas, but *Ashkenazy et al.* [1986] studied arrays of dipoles spaced at $0.7\lambda$, and obtained bandwidths of 20%. *Au et al.* [1996] studied arrays of patch antennas spaced at $0.52\lambda$, and concluded that bandwidths of >15%, with a VSWR< 2, could be attained over a scan angles of ±45 degrees. Here we assume similar results can be obtained for circular polarizations.

The beam could be steered lower, all the way to the horizon if desired, but efficiency would suffer considerably. This may not be a problem for routine spacecraft communications, which have considerable margin, but means that planetary radar or emergency spacecraft communications would be best if conducted when the target is high in the sky.

Another good reason to not point down to the horizon is the safety of nearby personnel. Even sidelobes could pose a problem here - with a facility of the size envisioned, a $-40$ dB sidelobe still represents a 1 MW EIRP. For safety, it seems safest to surround each quarter of the array with a tall metallic fence, such as used at Arecibo (though in this case the 15 meter fence was used to prevent the warm ground from radiating into the feed). Each quadrant has its own fence to allow unprotected personnel to service one portion of the array while the other portions remain in service.

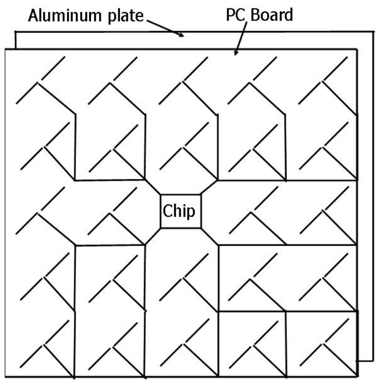


**Figure 2.** Board Design: This is a simplified schematic version, showing a 5 by 5 matrix of dipole antennas. The real board would have a 10 by 10 matrix of circularly polarized patch antennas. The board is spaced a fraction of a cm above an aluminum plate, which provides a ground plane for the antennas.

### 3.2. Design of the chip

The design of the chip involves several decisions. How many phase bins are required? How often must the phase be updated? Is amplitude control needed for each transmitter, or will phase control alone suffice? How does each chip receive its command and control information? How can the modulation of the many elements be coordinated?

First, fine grained phase control is not essential. If we have $N$ equally spaced phases, and pick the nearest one, then the error in phase must be in the range $-\pi/N - \pi/N$. The component of the E field in the desired direction shall be reduced by the cosine of the phase error. Assuming the errors are uniformly distributed (in general a reasonable assumption - see Appendix 2), the average loss in amplitude shall be

$$\frac{\int_{-\pi/N}^{\pi/N} \cos(\theta) d\theta}{2\pi/N} = \frac{\sin(\pi/N)}{\pi/N}$$

This loss is shown in Table 1. The point of diminishing returns is reached at 3 or 4 phase bits (8 or 16 phase possibilities). In what follows the use of 4 phase bits is assumed.

**Table 1.** Losses from different numbers of phase bins.

| Number of bits | Number of phase bins | Loss in dB |
|---|---|---|
| 1 | 2 | 3.92 |
| 2 | 4 | 0.91 |
| 3 | 8 | 0.22 |
| 4 | 16 | 0.056 |
| 5 | 32 | 0.014 |

One of the simplest ways of controlling the phase is to generate the 16 phases and then choose amongst them using a multiplexor. This basic technique has been demonstrated in a single chip phased array receiver at 24 GHz in the work of *Hashemi et al.* [2004] and should therefore not be a problem at 8 GHz.

The elements do not need variable amplitude for this application, since we are optimizing for maximum power at the beam center and not for beam shape or sidelobes. For a single beam, there is no penalty in EIRP for not controlling the amplitude. For generating multiple simultaneous beams the lack of amplitude control induces a small penalty. If an array is capable of generating a single beam of EIRP $P$, but is programmed to generate $M$ beams, adjusting only the phase but not the power of each element, then each beam will have an EIRP of slightly less than $P/M$. Appendix 2 shows this loss is, in theory, about 1.05 dB in the limit of large $M$. Simulations show a similar loss for smaller values of $M$.

### 3.3. Updating the phase

For almost all applications (with the possible exception of aiming a beam at a geosynchronous satellite) the phase of each transmitter module must vary as the Earth rotates and the target moves. Also, for spacecraft uplink, the motion of the target must be taken into account, so the signal arrives at the correct apparent frequency, and different frequencies must be generated for the different uplinks.

Any fixed or slowly varying phase offset can be compensated for by the command and control system. However, any practical command network would not have the bandwidth to command each transmitter phase change directly. Therefore each chip must be capable of locally computing at least some of the desired phase evolution. This leads to two related update rates. The first is how often the phase must be updated, which determines the required speed of on-chip calculations. The second is the rate at which the phase model must be updated, which determines the bandwidth needed between the central computer and the chips of the array.

Calculating the first rate is straightforward. For uplink applications, the phase update rate is determined by the need to generate different uplink frequencies from a common reference. Generating a frequency different by $f$ Hz, by slipping an $N$ phase transmitter, requires phase updates at $f \cdot N$ Hz. The deep space uplink band is 50 MHz wide[*JPL*, 2003]. Assuming a reference frequency in the center of the band, and 16 phase bins, this requires phase updates at a 400 MHz rate. For planetary radar applications, the required rate is simply the maximum modulation rate desired. The current limit is 20 Mbaud, but higher rates would be helpful on those rare occasions where the signal to noise permits, so perhaps 100 MHz would be reasonable target.

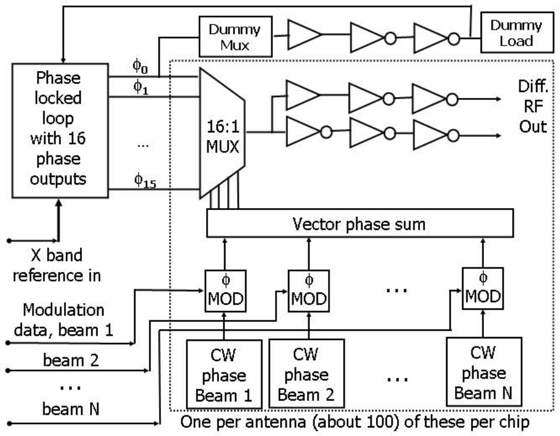


**Figure 3.** Design of Chip. This shows the RF path, and how the beams are modulated and summed.

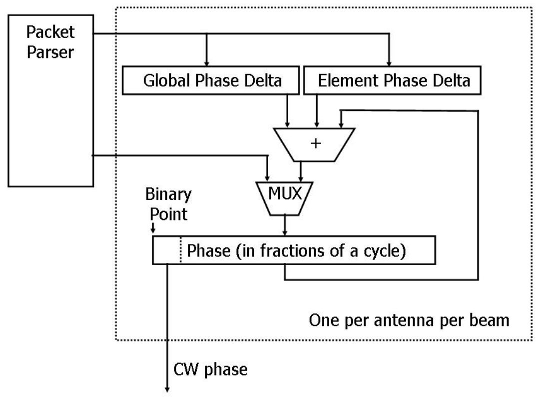


**Figure 4.** How the CW phase of each beam is computed.

### 3.4. Phase Model Updates

The second calculation is harder, since it depends on the model of phase evolution used. After a fixed phase, the next simplest phase evolution is linear in time. Assuming each chip could do such a linear evolution, how often do the coefficients need to be reloaded? Most motion in the solar system is circular, or nearly so, so this is the case we will analyze. We consider the case of a moving source; the moving target case is similar. For a single circular motion of angular velocity $\omega$ and radius $d$, the extra phase (in cycles) to be traversed is $P(t) = \frac{d}{\lambda}\sin(\omega t)$. We can expand this in a Taylor series about time $t_0$ to get:

$$P(t) = \frac{d}{\lambda}[\sin(\omega t_0) + \omega\cos(\omega t_0)(t - t_0) - \frac{\omega^2 sin(\omega t_0)}{2}(t - t_0)^2 + O(\omega^3(t - t_0)^3)]$$

Assuming we set the initial phase and rate of change at $t_0$, then the leading error term is

$$\frac{d}{\lambda} \cdot \frac{\omega^2 sin(\omega t_0)}{2}(t - t_0)^2$$

This error grows quadratically with time. A reasonable limit might be when it reaches 1/8 of a phase bin. If we have $N$ phase bins, using $|\sin(\omega t_0)| \leq 1$, the phase error is always acceptable if

$$|t - t_0| < \sqrt{\frac{\lambda}{4Nd\omega^2}}$$

The transmitted phase is determined by the sum of many of these (almost) circular motions. Even for a target fixed in inertial space there are 3 terms: the motion of the earth around the sun, the array center around the axis of the earth, and the orientation of the array around its center. For a target in orbit around another planet, there are two more terms - the orbit of the planet around the sun, and the orbit of the spacecraft around the planet. Table 2 shows how long a linear phase can be used with less than 1/8 phase bin error. A low orbit around Jupiter is assumed as this gives the fastest phase change of any likely solar system mission.

All terms except for array and tile orientation are shared by all elements of the array, and the sum of linear phase evolutions is itself a linear evolution. Therefore a single linear phase evolution, broadcast to all chips, will cover most of the needed phase variation.

The remaining components of phase evolution are tile and element specific, and are represented by the lines 'Tile orientation' and 'Array orientation' in Table 2. If both the phase and the phase derivative are set exactly at a given time, the errors in relative phase within a tile will will be less than 1/8 of a phase bin for the next 887 seconds. Likewise, they phase of a tile with respect to the array center will be good for 24 seconds. These times can be doubled by setting the error to zero in the middle of the time span instead of at the beginning. This rate is sufficiently slow that a simple command scheme, as shown in section 3.7, could keep all chips updated. A longer time between updates could be obtained with a higher order model, but is probably unnecessary. The hardware needed to locally compute a phase evolution linear in time is relatively straightforward, and shown in Fig. 4.

### 3.5. Distributing the phase reference

To generate a coherent signal, all chips in the array need a common phase reference, obtained by distributing a master reference frequency to every chip. Since an X-band wavelength is about 3.5 cm, we can only afford a few mm of variability in the effective lengths of the distribution network.

**Table 2.** Time in seconds until the phase evolution from a given motion deviates by 1/8 phase bin from a linear extrapolation.

| Motion | period | Radius (m) | Time (sec) |
|---|---|---|---|
| Tile orientation | 1 day | 0.15 | 887 |
| Array orientation | 1 day | 200 | 24.3 |
| Earth around sun | 1 year | 1.486e+11 | 0.325 |
| Earth spin | 1 day | 6.5e+06 | 0.134 |
| low Jupiter orbit | 3 hours | 7.2e+07 | 0.005 |
| Jupiter around sun | 11.9 years | 7.78e+11 | 1.693 |

Building such networks is a well studied problem [*Shillue*, 2002-Nov-14] and has well known solutions, since existing radio telescope arrays need to distribute even higher frequency signals over much larger distances. However, we wish to avoid the cost and complexity of the active length compensation schemes that achieve these high performances.

Fiber optics seem the best candidate for the master reference distribution. The coefficient of expansion of for optical cables is about $8 \cdot 10^{-6}$m/m/°K. Therefore if the array is 200 meters in radius, and we use uncompensated cables, then the temperature must held be constant to within 1°K. This is not practical if the cables are exposed, and the simplest solution is to bury them. The ALMA group estimates that the diurnal temperature variation of a cable buried 1 meter deep is about 0.001°K. This is sufficiently small that the stability is determined by the above-ground sections of the network. As long as these are kept to a few tens of meters, active length compensation of the feed network should not be required. This is the same conclusion as drawn by the Allen Telescope Array (ATA) [*Dreher*, 2003].

### 3.6. Modulating the beam

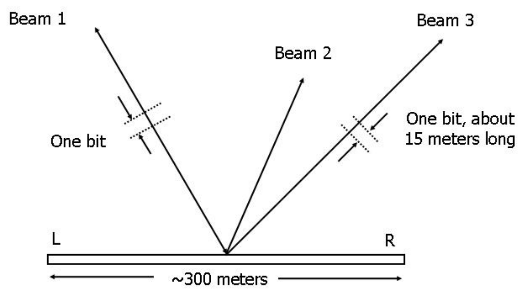


**Figure 5.** Array viewed from the side. Here the array is shown generating 3 beams. Although not to scale, this diagram shows that at the higher modulation rates, each bit is small compared to the physical size of the array.

Most applications of a deep space transmitter require a modulated beam. Normally phase modulation is used, and the necessary modulation rates depend upon the application. Radar needs the fastest rates to form high resolution radar images. Currently up to 20 MHz modulation rates are used [*Ostro et al.*, 2002], but this is limited by the bandwidth of the transmitters used and even faster rates may be useful in particularly close encounters. Spacecraft ranging requires modulation at rates of up to 1 MHz. Both radar and ranging use relatively simple sequences which could be computed on chip, and so do not require high data bandwidth to each chip. Uplink data, on the other hand, is completely arbitrary and must be sent to each chip in its entirity. Existing spacecraft uplinks typically run at a few tens of KHz [*JPL*, 2003], so a slow mechanism would suffice for current applications. However, it is anticipated that the uplink bandwith will be significantly greater in the near future, [*D'Addario*, 2005], so it seems prudent to allow for this in the design.

Since we are computing the desired phase for each beam already, phase modulation is straightforward. The modulators, shown in Fig 3, simply add a constant to the desired phase for a '1', and pass the phase unchanged for a '0'. If desired, another bit could be used to enable/disable each transmitter for each bit. This would double the bandwidth needed between the supervisory computer and the array, but would enable amplitude modulation, by turning off some or all transmitters for specified bits.

At the highest rates currently used, 20 Mbaud, each bit is 15 meters long when in flight. This is much smaller than the physical size of the array, so each transmitter must start to send the same data at a different time. See, for example, Fig. 5, where beam 1 is sent to the left, and beam 3 to the right. For the data of beam 1, the transmitter at R needs to run about 10-15 bits ahead of the transmitter at L, so that the modulated signals are all in phase in direction 1. For beam 3, the opposite is true - the transmitter at L must run ahead of the transmitter at R. The data bits, and the modulation, are exactly the same for each transmitter - only the start time is different. Therefore the data bits can be broadcast to all chips, but each chip must adjust the start time for each data stream. For any current or anticipated data rates, element specific delays are not needed - one delay per tile will suffice.

Delaying the bits until they are needed is not hard - a standard first-in first-out (FIFO) queue will work, and the central controller can keep each chip updated with the correct start time. The hardest part is that each chip must have a local clock, good to about a nanosecond relative to all other clocks in the array. Since they all receive a common reference frequency, maintaining synchronization is easy. Establishing synchronization on startup is harder, but seems possible. For example, the central processor could flip the phase on the reference, and each chip could set its clock to 0 when it sees a phase flip. The the central computer can send each chip a chip-specific offset to be applied to the clock.

### 3.7. Commands to the chips

A possible command structure is shown in figure 6. Each chip receives packets from below and from the left, and copies them to the chips above and to the right. In the absence of faults, both incoming packets will be identical. Each packet includes a checksum; if the incoming packets are not identical, the chip looks for one with a correct checksum, and sends it to both right and above neighbors. This scheme means each chip gets each packet even if there are faulty chips in the array (with a few minor exceptions - the first chip must work, and we assume that faulty chips don't generate erroneous packets with correct checksums). Assuming a 64 MHz bit rate, and 128 bit packets, each chip takes 2 usec to get each packet. If the array is divided into sections of no more than 500 by 500 tiles, a command wold take about a millisecond reach every tile in the array. This scheme implies some chips will get the data to be sent well before others. If a beam is transmitting at 20 MBaud, for example, it may need to buffer 20,000 bits to allow the data to reach all other tiles.

Using the numbers above, the control overhead is tolerable. Updating each of 250K chips once each 45 seconds for 16 beams implies one update every 11 usec, or about 20% of the capacity used for phase control. The remaining packets can be used for data, for an aggregate data rate in the range of 50 Mbits/sec. With this architecture, the data rate is shared among all the beams, but this should be little problem in practice since the deep space uplink band is only 50 MHz wide. Unless substantially more complex modulation schemes are adopted, 50 Mbits/sec will suffice. The one application that could use higher rates is planetary radar, but in this case pseudo-random codes are normally used. In this case, each chip could compute the sequence itself, requiring very little communication.

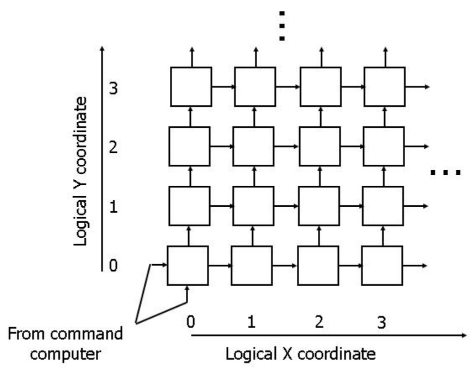


**Figure 6.** Commanding the array of chips. Note that the logical index (row and column) does not map in a simple way to exact physical locations, since the tiles may be placed non-uniformly, and the array may not be perfectly flat.

A relatively small number of different types of commands are needed to drive the array. Initialization commands must set the logical (row/column) address of each chip, and set the clock. Chip specific commands set phase models, and bit start times. Broadcast commands send data, data rates and modulation information, and the portion of the phase model that applies to the array as a whole.

### 3.8. Calibration

The uncalibrated transmitter will have a stable (but unknown) phase shift to each element. Likewise, the exact location of the phase center of each element is also unknown. The purpose of calibration is to determine these values. Conventional radio telescopes solve this problem by observing a variety of far away sources (in different directions) and then adjusting the coordinates and phases until the correlations are maximized. We could do the corresponding operation (observe the transmitter from a variety of different and distant locations, and record the phases sent by each element), at least for the initial calibration. Here is one possible procedure. First, enable a subset of the transmitters in the array, each to tuned to a unique frequency. Then overfly the array with a plane, and record the detailed waveform received over the entire bandwidth. Assuming the phase of the received waveform can be measured to 10% accuracy, the phase and location of each transmitter (and the trajectory of the plane) can be reconstructed to mm accuracy. How many transmitters can be calibrated in a single pass? Assuming a small plane is used, to keep costs low, we might pick an altitude of 1 km and velocity of 20 m/s, taking 100 seconds to pass through the portion of the beam within 45 degrees of the zenith. The maximum doppler shift at X-band will be less than 500 Hz, so a frequency spacing of 5 KHz will suffice to enable each to be measured independently. If the array can cover the whole X-band uplink of 50 MHz, then at least 10,000 elements at a time, so 100 passes should suffice for a 1M element array. This technique, though expensive and inconvenient, also allows the signal strength and polarization to be measured as a function of zenith angle.

For more frequent but less thorough calibrations, a simpler scheme might suffice. A receiver mounted high and nearby (perhaps on a tall pole) could measure the phase differences between sets of transmitters. Although this location is not in the primary beam of each antenna, it should suffice for tracking changes.

How often should the array be calibrated? Presumably, all transmitters in a single chip will track, and the coordinates of all the antennas should vary slowly at most. Temperature measurements of components of the array might allow much of the variation to be compensated. The ATA has similar problems (though it receives rather than transmits) and has concluded that relatively infrequent calibration, perhaps every few hours, will suffice [*Dreher*, 2003].

Since a few modules can be off line at any given time with little impact on performance, the array could be calibrated while in service. For example, the phase difference between any two transmitters can be measured by setting those two transmitters to any unused frequency, then alternating between them and measuring the phase difference. This would have a negligible effect on the array operation, and since every pair could be measured if necessary, phase closure should be straightforward.

## 4. A possible proposal

Here are a few examples that show, very roughly, what might be possible. The two main applications are DSN uplink and planetary radar. To fit the existing infrastructure, the DSN application should run at 7.2 GHz (4 cm) and the radar at 8.6 GHz (3.4 cm). Although a compromise design that could do both might be possible, efficiency in at least one of the two modes would surely suffer. Also, such an array could not be used for both applications at the same time, since the instantaneous bandwidth does not support it, and hence would have many of the same scheduling problems as exist today. Therefore we assume that each is built and optimized separately for its intended application. The planetary radar requires only one beam, but as strong as possible. DSN uplinks today at X-band have EIRPs of 0.4 TW from the 70 meter antennas and 0.1 TW from the 35 meter antennas. The proposed array would be most useful if it could replace all current uplinks and allow for future expansion, and do so even if a significant fraction of the array was down for maintenance. The full array is therefore sized to support one 0.4 TW uplink and 15 0.1 TW uplinks, for a total EIRP of 2.0 TW.

Four possible sizes are listed in table 3.

Spacing the array elements $0.71\lambda$ apart allows pointing 45 degrees from the zenith. We further assume the individual antennas have roughly a 3 dB gain (the circularly polarized patch antennas found in the literature have about 6 dB gain at zenith, dropping to 3 dB at 45 degrees). We also assume each board would have 100 array elements, and that each chip contains 100 drivers, so there will be one chip per board. For the purpose of the following discussion, we assume that a tile costs $20. (The basis behind this estimate is explained in Appendix 1.) In particular, the low chip cost is reasonable since it can almost certainly be built with a conventional $0.13\mu$ or smaller CMOS process [*Doan et al.*, 2004; *Franca-Neto et al.*, 2004]. Power dissipation of the chip should not be a big stumbling block. At 40% efficiency for the power amplifier stages, and neglecting the other portions of the chip, the chip would need to dissipate 1.5W for 1W total RF output. (40% should be achievable since 42.6% has been reported at 5.7 GHz, at higher power levels and with an older process [*Heydari and Zhang*, 2003].) The package would require 200 pins for the differential RF outputs, and perhaps 50 more pins for power, ground, and control. We allow for 2 dB of miscellaneous losses, due to the use of low cost package and board technology. DC power supplies are assumed to cost 0.5$/watt. We also assume the concrete slab costs $50 per square meter, and that each board can be installed on the slab in 15 minutes, and labor costs $25 per hour. This task is most uncertain and the most susceptible to tradeoffs. Embedding the power, RF, and command distribution in the slab would make the slab more expensive but the installation easier.

The column labeled 'Existing' has roughly the performance of the existing radar transmitters. It would cost about \$9 million to build, much less than the cost of one new 34 meter dish for the Deep Space Network [*JPL*, 2002]. The largest model, still significantly less expensive than a 100 meter antenna, would have about 30 times the EIRP of any existing transmitter. This would allow study of the rings and moons of Uranus, and enable a quite detailed study of any main belt asteroids. Note that unlike single dish transmitters, there is no technical reason preventing even larger transmitters from being built, if desired.

**Table 3.** Different possibilities for transmitter sizes. For comparison, the largest existing transmitters are Goldstone, with an EIRP of roughly 10 TW at any elevation, and Arecibo, with an EIRP of roughly 20 TW within 20° of the zenth.

| | DSN | Existing | Bigger | Large |
|---|---|---|---|---|
| Wavelength (cm) | 4.0 | 3.5 | 3.5 | 3.5 |
| Number of beams | 16 | 1 | 1 | 1 |
| Element spacing (cm) | 3.0 | 2.5 | 2.5 | 2.5 |
| Tile size (cm) | 30 | 25 | 25 | 25 |
| Elements per sq. meter | 1100 | 1600 | 1600 | 1600 |
| Number of chips | 125K | 280K | 750K | 2M |
| EIRP (45° elevation) | 2 TW | 10 TW | 74 TW | 354 TW |
| EIRP (zenith) | 4 TW | 20 TW | 148 TW | 708 TW |
| Cost (millions of \$) | 5 | 9 | 24 | 63 |

## 5. Usage

With a flexible and dedicated transmitter, asteroid research in particular could be carried out much more efficiently. With no transmitter constraints, the receiver would be freed up for use over the entire allocated telescope time. It could acquire one observation, re-point, get another echo, and so on. This would be much more efficient than the current system even if asteroid research was allotted only the current fraction of telescope time.

If combined with a low cost receiving array (such as the ATA, estimated to cost \$28M) [*SETI Institute*, 2002], a dedicated asteroid research facility could be built for less than \$50M. With a dedicated facility, no asteroid need ever be 'lost', since they can be targeted by radar as soon as they are discovered. Even a single radar observation vastly improves an optically determined orbit since the radar measurements are largely orthogonal to optical measurements - optical measurements estimate angles but not distance, while radar is best at estimating range and range rate.

A more flexible transmitter would also help Arecibo in particular. Arecibo can only observe targets out to the range of Saturn if it must function as both a transmitter and receiver. This is because it has a limited field of view (about 20 degrees around the zenith). Thus a given target is in the field of view for a maximum of 2 hours, and that only in the most favorable circumstances. Since the round trip light time to Saturn is about two hours in the best of cases, Saturn and its moons can only be imaged in the best circumstances, which occur only every few years. Targets further away cannot be imaged at all. A transmitter with a wider field of view could allow Arecibo to probe Saturn regularly, perhaps probe further targets, and in general be more productive.

For spacecraft command communication, eliminating the need for each receiving antenna to simultaneously transmit allows a helpful gain in receive efficiency. The noise figure can be improved by almost a factor of 2 in some cases [*JPL*, 2003][*Britcliffe1 and Fernandez1*, 2001]. For the JPL 70 meter antennas, at S-band, the normal diplexed antennas achieve a 17°K noise temperature, but a special receive-only modification built for Galileo achieved a noise temperature of 9°K.

## 6. Implications for SETI

One of the main ways the search for extraterrestial intelligence is conducted is by looking for microwave signals from other civilizations. Such efforts have been made, but even with large radio telescopes a TW or larger EIRP would be required for detection[*Tarter*, 2001]. Since the days of Cyclops [*Oliver et. al.*, 1971] (the seminal SETI roadmap) this signal has been assumed to be generated by a high power, omni-directional, microwave beacon. However, it is a big assumption to expect another civilization, even if advanced and altruistic, to devote roughly the entire electrical output of the current Earth civilization to drive an omni-directional beacon in the hopes that someone, somewhere, is listening.

But an ommi-directional beacon may not be needed. An advanced civiliation may well be able to narrow down the list of likely targets. (Perhaps they only target stars with planets, a distinction we ourselves are beginning to make). A beacon built with the technology proposed here could illuminate a few hundred stars with TW signals for a few tens of M\$ and a few MW of power. This is easily within the reach of corporations and religious groups, who may be more motivated to transmit than governments. Therefore, if we ever do receive a signal, it may well consist of advertising or attempts at religious conversion, rather than words of wisdom from some galactic government.

## 7. Future Research

The basic technology needed for such a transmitter is not in doubt. There are many existing phase array transmitters, and all the elements needed to integrate the phase shifters and amplifiers have been demonstrated. The main uncertainty is cost. While the estimates are based on existing industrial experience (see Appendix 1), significant uncertainties remain. Three issues in particular stand out - array element spacing, low cost IC packaging, and reference frequency distribution. First, the element spacing affects almost all aspects of the array. A detailed analysis of element spacing versus cost and performance is called for. Next, low cost packaging for the IC *must* be used - even a \$10 package would increase the budget by a factor of 1.5. We have assumed that the low power levels, narrow band operation, and generous margins for losses will allow us to compensate for very cheap packages, but this assumption must be verified. Finally, propagating the reference frequency from the (presumably) fiber optic distribution network to each chip needs further investigation. These issues, and a detailed design for physically mounting and connecting the PC boards, should be verified in detail before any mass production of components is initiated. Fortunately, a small number of the needed components could be combined into a small array, and evaluated in detail, before volume production is begun.

The future for this technique looks very promising. As chip geometries continue to shrink, the same techniques work at even higher frequencies. This allows more antennas on a board, for potentially higher EIRPs. For example, increasing the frequency from 8 to 32 GHz would allow 16 times as many antennas on a PC board of the same size and hence the same cost. If the chip technology, and particularly the pin count of cheap packages, keeps up, then this would allow 16 times the EIRP assuming the total power per board remains constant.

## 8. Conclusions

It seems quite possible to build large phased array transmitters for deep space applications. Compared to the existing techniques of large transmitters and large dishes, these offer several advantages - lower cost, higher reliability and availablity, more flexibility, and potentially higher EIRP.

## Appendix 1: More detail on costs

Is a cost of $20 per tile remotely feasible? The final proof is in the detailed design and quotes from vendors. Without a detailed design, any cost estimated must be derived from extrapolation from, and analogy with, existing components. In addition the savings from high volume production remain uncertain. Therefore all costs in this section are quite speculative. In particular, if any of the major assumptions - $0.71\lambda$ spacing, low cost packaging, cheap optical receivers - fails badly on detailed analysis, the cost could easily double.

A design that minimizes the total cost is difficult to determine, since all the major elements (board, package, and chip) interact. For example, PC boards are usually cut from 300 mm (actually 12 inch panels), so the PC board component would be cheaper if that size board was used. But (for the planetary radar) this would mean increasing the number of drivers per chip to 144. This in turn would increase the power dissipation, and require more pins, which may well lead to a more expensive package. In addition the lines to the antennas would now be longer, and a lower loss (and hence more expensive) substrate might be required.

Interactions such as this mean minimizing costs requires the parallel design of the PC board, the package, and the chip. Fortunately the design is conceptually simple and this should be practical. Furthermore all elements could be prototyped before volume production must start, so the risk of such an integrated approach should be manageable.

### PC board, backing plate, and spacers

Each tile is comprised of an aluminum plate, a PC board, and the spacers that keep them apart. The planetary radar tile proposed in this article is 25 by 25 cm. Thin (0.032 inches, or a little less than 1 mm) aluminum plate sells for about $10 per square meter, or $0.63 per tile. Small spacers are a staple of the personal computer industry, and quite inexpensive. Therefore $1 per tile is allocated for the aluminum backing plate and the spacers.

High volume PC board production is a routine industrial task. The cost of a board depends on the materials used, the number of layers, the tightness of the tolerances, the delivery schedule, and many other details. For the most common types of printed circuit boards, cost data is widely available. Table 4, from `http://www.eeinternational.net`, shows estimated production costs for boards of different types and volumes.

**Table 4.** Costs of printed circuit boards of different types and volumes.

| Quantity (sq. ft) | Single Sided (per sq. inch) | Double Sided (per sq. inch) | 4 Layers (per sq. inch) |
|---|---|---|---|
| 1,000 | $ 0.14 | $ 0.18 | $ 0.25 |
| 3,000 | $ 0.10 | $ 0.12 | $ 0.20 |
| 5,000 | $ 0.08 | $ 0.10 | $ 0.18 |
| 10,000 | $ 0.07 | $ 0.09 | $ 0.16 |
| 20,000 | $ 0.06 | $ 0.08 | $ 0.14 |
| 30,000 up | Contract | Contract | Contract |

What PC board characteristics are required? We need at least two layers, since we need to form stripline (transmission line) routing from the chip to the antennas, to keep a controlled impedance. We want no more than two layers, both because the cost is greater, and because extra layers cannot be used for most of the board area (since we are using it as an antenna). The only place where more layers could possibly be used is directly under the chip, and we should strive to use only two layers here as well. Although routing all needed signals in two layers is often difficult, in this case it should be possible since all the routing is planar, and all the chip outputs are balanced.

In terms of materials, the furthest antenna is roughly 170 mm (about 6.6 inches) from the center of the board. We need less than 2 dB of losses in this length to meet our spec.

The highest volume explicitly mentioned in the table (20,000 square feet) gives a cost for a double sided board of $0.08 per square inch, which would result in a cost of $7.76 per PCB. This estimate cannot be used as is, however. One the one hand, prices are still dropping with increased volume, and we need at least an order of magnitude more boards than is covered in this table. Each previous order of magnitude in volume drops the price in half. If this trend continues, it would imply a cost of about $4.00 per board.

However, these prices are quoted for the most common PCB material, called FR-4. This material is relatively lossy in the microwave region, and a more expensive material will be needed. Driven largely by the communications market, there are a large number of alternative board materials available with a wide variety of performance/cost tradeoffs. For example, a paper by *McCarthy and Morell* [2003] discusses the manufacture of boards with a composite material composed of part PTFE (Teflon) for electrical performance and part FR-4 for mechanical performance and compatibility with existing processes. This material is very close to meeting our requirements - the losses are about 2.33 dB for the longest lines. For high volume production of boards with this material, they conclude "Preliminary data suggests the laminate material will cost 4.0x FR4 and finished printed circuit boards will cost OEMs 1.6-1.9x FR4, depending on volume". Applying this multiplier to the costs above yields a total board cost of a little under $8 per board. Overall, a $10 price per tile in high volume seems possible.

### The chip

There are three main components to the chip cost. The chips must first be built, then each chip must be tested and the good ones selected, and then these good parts must be put in packages. Volume is not a problem - a million or so chips at a few dollars each is well within the range of normal industrial practice. Note that Bluetooth chips, which combine digital logic and a 2.4 GHz transmitter and receiver, sell for less than $5 [*Souza*, 2002] in quantities of millions *per week* [*Bluetooth Special Interest Group*, 2003].

We need 100 differential RF outputs. Since all outputs shall be on all the time, the current drain is relatively constant, and the total chip draw is only about 3 watts. 20 each of power and ground pins should be sufficient. With a few pins for command and control, perhaps 256 pins would be required. If the inputs and output (IOs) are arranged in two rows around the periphery, and assuming each is 120 microns wide, then the chip must be at least 4 mm on a side in order to accomodate the IOs. The output drivers described in *Farzan and Johns* [2004] nearly meet the requirements. They have differential output, operate at 10 Gbits/sec (5 GHz), and occupy 0.16 $mm^2$ in a $0.18\mu$ CMOS technology. Scaling to $0.13\mu$ would give 0.83 $mm^2$ and sufficiently fast operation. 100 of these drivers would occupy 8.3 $mm^2$ or about half of a 4 x 4 mm chip. Using public figures for a 130 nm process [*UMC*, 2004] ( 220K gates/$mm^2$ and 400K bits of memory/$mm^2$ ), the required digital logic should fit in the center of a chip with little problem. The

16 phase oscillator, by far the largest analog component in the core, should fit in 1 $mm^2$, by analogy to a similar oscillator*Hashemi et al.* [2004]. These chips could be built in a standard CMOS process [*Doan et al.*, 2004; *Franca-Neto et al.*, 2004], which costs about $1000 per 200 mm wafer processed [*Clendenin*, 2004]. About 1800 chips of this size would fit on a 200 mm wafer, and since the chips are small, the yield should be good (Bluetooth transceiver chips, a similar part, expect yields of 80-90% [*Koupal et al.*, 2003]). A cost of $1/part seems possible for fabricating the silicon.

The next question is test costs. The digital part of the chip is quite conventional and should be easy to test. The analog portion consists of the phased lock loop, the phase generation, and the output drivers. With care, the basic functionality of the output drivers could probably be determined with digital measurements. The remaining analog parts such as the phase locked loop (and the RF characteristics of the output drivers, if those are required) would have to be evaluated with analog measurements. Perhaps the most similar chips in volume production are the various Bluetooth tranceivers, which also combine a moderate amount of digital logic with microwave analog functionality. These parts are built, packaged, and tested for roughly $1.50 each to support their $5 selling costs, and the test costs account for no more than $0.50 of this [*Ozev et al.*, 2002, 2001]. Though we need to test at a higher frequency, and our volumes are less, it seems reasonable to assume that we can test out chips for no more than $2 each.

The final cost component is the package, where we assume a cost of about 1 cent/pin [*SEMATECH*, 2004]. This is a low cost package, not optimized for microwave design, but we assume that we compensate for the parasitics since we are only operating at a single frequency. At most 300 pins would be required, so about $3.00 should be reserved for the package cost.

Overall a chip cost of $6.00, composed of $1 fabrication cost, $2 test cost, and $3 package cost, seems possible in volume production.

**Clock, power, and data**

Each tile must get power to the chip, communicate with the neighboring tiles, and receive the master reference frequency. The power and data connections can choose from many existing technologies, with very conventional requirements. Perhaps $1 per tile should be allocated for these connections.

The conceptually simple task of distributing the clock is one of the most difficult tasks to do cheaply. The most straightforward method would be to modulate a central laser at the reference frequency, drive it through a network of optical splitters onto a fiber for each tile, and convert from fiber to electrical on each tile. This would surely work, but is too expensive currently. For example, photodiodes that will work at this frequency cost roughly $10 each[*Xponent*, 2003], not including assembly. On the plus side, most optical communications receivers are specified to work at −20 dBm, so only a few watts of optical laser power should suffice for the array.

The market for high speed optical components is changing rapidly as applications such as high speed optical networking and fiber to the home (FTTH) are developed. There are also many other engineering options - distribute a lower frequency reference using less expensive parts, then multiply it up on chip, or deliver the signal to the boards using waveguides or cables, using fibers only for the long distance transport, and so on.

For the cost estimates we assume that one of these methods is viable and can be implemented for $3 per tile.

## Appendix 2: No amplitude control loses 1.05 dB for multiple beams

This appendix contains the math behind creating multiple beams from a phased array transmitter. We compute the tradeoff between number of transmitters, number of beams, and beam quality. We find the expected amplitude, power, phase noise, and determine the degradation if all transmitters have equal and fixed power.

**Rayleigh's random walk**

We use a result first derived by *Rayleigh* [1880]. If we start at the origin, and make $n$ steps of unit length in random directions, then the odds of finding the endpoint between $r$ and $r + \mathrm{d}r$ from the origin is:

$$\frac{2}{n} r e^{-r^2/n} \mathrm{d}r$$

. The mean squared displacement is $n$, leading to the well known result that the average displacement grows like $\sqrt{n}$. The $X$ or $Y$ component, considered alone, has mean 0 and variance $n/2$.

**Intro**

Assume that we have $N$ transmitters, and wish to generate $M$ beams. Furthermore, assume that each transmitter has an output power of 1 unit (either fixed at this value, or averaged over the whole array, depending on the model). Assume that the beams are randomly located on the sky, and assign the phases and amplitudes of each transmitter as follows. For each target, compute the desired phase at the transmitter. Then for each transmitter, sum (as vectors) the desired phases for each target, with amplitude $1/\sqrt{M}$. In the variable amplitude model this determines both the amplitude and phase for each element. In the constant power model we keep the phase but set the amplitude to 1. In the variable amplitude case the power per transmitter will average 1, by direct application of Raleigh's result above.

**Results with variable power transmitters**

We look at the result as seen by one receiver. To do this, we add transmitter voltages with the phase shift appropriate for the receiver's direction. The $1/\sqrt{M}$ contribution computed from that receiver will add up in phase. The other $(M-1)N$ contributions shall appear as a sum of random vectors, and shall have magnitude approximately $\sqrt{\frac{(M-1)N}{M}}$. Thus the final voltage in the phase plane as seen by that receiver is shown in Figure 7:

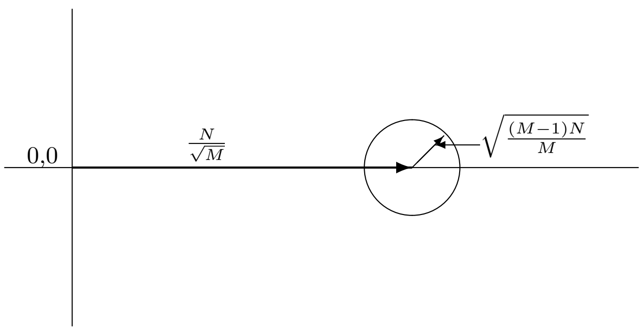


**Figure 7.** How the voltages add up

Although drawn as a circle, the error is really a cloud. All we know is that the voltage vector ends somewhere in that area. Therefore the main characteristics of a beam are as follows:

The expected amplitude $A(M,N)$ is

$$A(M,N) = A_0 \frac{N}{\sqrt{M}}$$

where $A_0$ is the amplitude induced by a single transmitter. Equivalently the EIRP per beam is

$$\mathrm{EIRP}(M,N) = P_0 \frac{N^2}{M}$$

where $P_0$ is the EIRP of a single transmitter. To find the expected variance in voltage, we want only the X component of the cloud. This will be $\frac{1}{M}\frac{(M-1)N}{2}$, and the standard deviation will be the square root of this. The Y component alone will have the same variance. The relative uncertainty in amplitude yields the signal to noise ratio.

$$\mathrm{S/N}_{\mathrm{power}}(M,N) = \frac{M-1}{2N}$$

and the phase noise (in radians) will be

$$\mathrm{PhaseNoise}(M,N) = \sqrt{\frac{M-1}{2N}}$$

**Theory and Experiment**

A simple program was written to test this model, and allows the user to experiment with $N$ transmitters and $M$ beams. The test case was 2 dimensional, which should make no difference. Each transmitter had an amplitude of 1 in the constant power case, and an average squared amplitude of 1 in the variable power case. We assume all the targets are far enough away so that the angles as seen by all transmitters are identical. The transmitters are randomly located between $-N/2$ and $N/2$ meters at an average density of 1 per meter. The beam angles are randomly chosen between $\pm 45$ degrees of the zenith. The wavelength is randomly chosen to be 0.0567 . The results are shown in Table 5.

The agreement of theory and experiment for large $M$, values 100 or greater, is excellent. The agreement for small $M$ is not as good, though this is to be expected. The real distribution is not per Rayleigh for $M$ as small as 10 (his solution is valid only for large $M$), and the distribution cannot be measured well from few samples. The overall performance, however, is fine for the applications considered here.

**Constant power transmitters**

It complicates the design considerably if each transmitter has to be able to control its amplitude as well as its phase. How much do we give up with constant amplitude? First, we need to find the expected amplitude in the desired direction at each transmitter. We assume without loss of generality that the desired signal has phase 0, and treat the remaining $M-1$ vectors as a random walk. We get a diagram that looks like Figure 7, except that the displacement from the origin is $1/\sqrt{M}$ and the circular error region now has radius $\sqrt{\frac{M-1}{M}}$, and is now much larger than the distance to the origin. This can be thought of as a random walk of radius nearly 1 (actually $\sqrt{\frac{M-1}{M}}$) with the origin shifted slightly by $1/\sqrt{M}$. Note that this distribution, and the distribution after clipping, depends only on the value of $M$; the variation with $N$ is taken care of by summing $N$ of these distributions.

Keeping the phase, but making the power constant, is the same as mapping every point in the phase plane along a given radial onto the single point where that radial intersects the unit circle. In other words, if the point is expressed in polar coordinates as $(r,\theta)$, we map it to $(1,\theta)$. We can treat this as follows.

The random part of the distribution consists of M-1 steps of size $1/\sqrt{M}$. From Rayleigh, the odds of finding this in the ring between distance $r$ and $r + \mathrm{d}r$ is

$$\frac{2M}{M-1} e^{-r^2 \frac{M}{M-1}} r\mathrm{d}r$$

To get the area density, we divide by the area of the ring to get

$$\mathrm{Density}(r) = \frac{1}{\pi}\frac{M}{M-1} e^{-r^2 \frac{M}{M-1}}$$

We want the expected value of the X component of the resulting vector, compared to the value of $1/\sqrt{M}$ that we get in the variable power case. This is computed by the following equation.

$$\frac{\int_0^{2\pi} \cos(\theta) \lim_{M\to\infty} \left[\frac{1}{\pi}\frac{M}{M-1}\int_0^{\infty} e^{-(r(q,\theta))^2 \frac{M}{M-1}} q\,\mathrm{d}q\right] \mathrm{d}\theta}{1/\sqrt{M}}$$

Where $r(q,\theta)$ is the distance from the center of the distribution

$$r(q,\theta) = \sqrt{(q\cos\theta - 1/\sqrt{M})^2 + (q\sin\theta)^2}$$

The part of the formula in square brackets is the probability that the phase is within $[\theta, \theta + \mathrm{d}\theta]$. This turns out to be a complicated expression which can be simplified provided we consider only large $M$, which explains the limit. The outer integral simply averages over all angles, taking the contribution to the X component times the probability that angle is found. Finally, we take the ratio to $1/\sqrt{M}$ since that is the expected value in the variable power case.

We start by expanding the expression for $r$ and using $\sin^2\theta + \cos^2\theta = 1$ to get

$$r^2(q,\theta) = q^2 - \frac{2}{\sqrt{M}} q\cos\theta + \frac{1}{M}$$

and so the term in square brackets becomes

$$\frac{1}{\pi}\frac{M}{M-1}\int_0^{\infty} e^{-(q^2 - \frac{2}{\sqrt{M}} q\cos\theta + \frac{1}{M})\frac{M}{M-1}} q\,\mathrm{d}q$$

Since we are only interested in $M$ large, we can drop the terms of $1/M$ which are the square of the terms with $1/\sqrt{M}$. To this order, $M/(M-1) = 1$, so we can drop these terms as well, to get

$$\frac{1}{\pi}\int_0^{\infty} e^{-(q^2 - \frac{2}{\sqrt{M}} q\cos\theta)} q\,\mathrm{d}q$$

which can be re-written as

$$\frac{1}{\pi}\int_0^{\infty} e^{-(q^2)} e^{\frac{2}{\sqrt{M}} q\cos\theta)} q\,\mathrm{d}q$$

We expand the second exponential into a power series and keep only the first term to get:

$$\frac{1}{\pi}\int_0^{\infty} e^{-(q^2)}(1 + \frac{2}{\sqrt{M}} q\cos\theta) q\,\mathrm{d}q$$

**Table 5.** Randomly located transmitters, random beam directions, variable power transmitters

| M | N | Amplitude | | Ampl. Variation | | Phase Variation | |
|---|---|---|---|---|---|---|---|
| | | Theory | Exp | Theory | Exp | Theory | Exp |
| 10 | 500 | 158.1 | 154.2 | 15 | 9.85 | 5.44 | 5.05 |
| 10 | 5000 | 1581 | 1531 | 47.4 | 73.9 | 1.71 | 1.93 |
| 10 | 50000 | 15811 | 15772 | 150 | 177 | 0.54 | 0.47 |
| 10 | 500000 | 158114 | 158231 | 474 | 438 | 0.17 | 0.14 |
| 100 | 5000 | 500 | 496 | 49.75 | 50.04 | 5.70 | 5.58 |
| 100 | 50000 | 5000 | 4979 | 157.3 | 159.6 | 1.80 | 1.82 |
| 100 | 500000 | 50000 | 49928 | 497.5 | 496.1 | 0.57 | 0.53 |
| 1000 | 50000 | 1581 | 1578 | 158 | 161 | 5.72 | 5.65 |
| 1000 | 500000 | 15811 | 15826 | 500 | 646 | 1.81 | 1.86 |

The alert reader (if any) might well ask if this is legitimate, since the dropped terms are multiplied by powers of $q$, which ranges to infinity. However, the first term, $e^{-q^2}$, tends to 0 faster than the missing terms grow, and as $M$ increases the contribution of the dropped terms becomes arbitrarily small compared to the terms we have kept. This gives us

$$\frac{1}{\pi}\int_0^\infty qe^{-q^2}\,\mathrm{d}q + \frac{1}{\pi}\frac{2\cos\theta}{\sqrt{M}}\int_0^\infty q^2e^{-q^2}\,\mathrm{d}q$$

The first term will not contribute to the outer integral, since it has no dependence on $\theta$ and hence integrates to 0 over one whole cycle. This leaves

$$\frac{1}{\pi}\frac{2\cos\theta}{\sqrt{M}}\int_0^\infty q^2e^{-q^2}\,\mathrm{d}q$$

This is a well known definite integral, value $\sqrt(\pi)/4$, leading to

$$\frac{1}{\sqrt{M}}\frac{\cos\theta}{2\sqrt{\pi}}$$

This is the non-uniform part of the probability that the phase is $\theta$. Now we integrate over all phases. If the phase is $\theta$, the contribution to the phase sum is $\cos\theta$, so we multiply the contribution times the probability of that contribution to get

$$\frac{1}{\sqrt{M}}\frac{1}{2\sqrt{\pi}}\int_0^{2\pi}\cos^2\theta\,\mathrm{d}\theta$$

but $\cos^2\theta = 1/2 + 1/2\sin(2\theta)$ has an average value of 1/2, so the last integral has a value of $\pi$, leading to an expected output voltage of

$$\frac{1}{\sqrt{M}}\frac{\sqrt{\pi}}{2}$$

We compare this to the expected value with no clipping, $1/\sqrt{M}$, to get the final relative amplitude of

$$\frac{\sqrt{\pi}}{2}$$

To check the previous calculation, we can also evaluate the clipping process numerically, and compare this to the analytical result with no clipping. We do this for various values of $M$, as shown in Table 6.

**Table 6.** Results of numerical simulation

| M | multiplier |
|---|---|
| 4 | 0.94603 |
| 8 | 0.915935 |
| 16 | 0.901281 |
| 32 | 0.894021 |
| 64 | 0.890361 |
| 128 | 0.888486 |
| 256 | 0.887506 |
| 65536 | 0.886277 |
| 1048576 | 0.886242 |

It certainly appears reasonable that this value is converging to $\sqrt{\pi}/2 \approx 0.8862269$ for large enough $M$, and the rate of convergence is right ($1/\sqrt{M}$ is about 1000 for the largest cases, and the result differs from the analytical limit by roughly 1 part in 1000). This confirms $\sqrt{\pi}/2$ as the expected value, and so the EIRP is about $\pi/4$, or about 0.7853 relative to the expected value of the variable amplitude case. This is a loss of approximately 1.049 dB.

So the final EIRP per beam is

$$\mathrm{EIRP}(M,N) = \frac{\pi}{4}P_0\frac{N^2}{M}$$

where $P_0$ is the EIRP of a single transmitter. To find the expected variance in voltage, we now have the sum of the X component of $N$ unit vectors oriented at random. This variance will be $0.5N$, and the standard deviation will be the square root of this. The Y component alone will have the same variance. So the signal to noise, expressed in terms of power, is

$$\mathrm{S/N_{Power}}(M,N) = \frac{4}{\pi}\frac{M}{2N}$$

and the phase noise (in radians) will be:

$$\mathrm{PhaseNoise}(M,N) = \frac{2}{\sqrt{\pi}}\sqrt{\frac{M}{2N}}$$

Comparing these predictions to experiment gives the results shown in Table 7. Once again the agreement is excellent for large $M$, and adequate for small $M$, as expected.

### Evenly spaced transmitters

What if the transmitters are evenly spaced, as in the proposed design? Then the phases will presumably be more correlated, since for each direction there are just two random variables (initial phase and delta) instead of $N$. The experiment shown in Table 8 shows the effect of even spacing

**Table 7.** Variance of the generated beams

| M | N | Amplitude | | Ampl. Variation | | Phase Variation | |
|---|---|---|---|---|---|---|---|
| | | Theory | Exp | Theory | Exp | Theory | Exp |
| 10 | 500 | 140.1 | 137.4 | 15.8 | 12 | 6.46 | 8.16 |
| 10 | 5000 | 1401 | 1388 | 50 | 53 | 2.04 | 2.09 |
| 10 | 50000 | 14013 | 14086 | 158 | 208 | 0.65 | 0.47 |
| 10 | 500000 | 140125 | 141057 | 500 | 458 | 0.20 | 0.16 |
| 100 | 5000 | 443 | 441 | 50 | 51.6 | 6.47 | 6.15 |
| 100 | 50000 | 4431 | 4427 | 158 | 157 | 2.04 | 2.15 |
| 100 | 500000 | 44311 | 44310 | 500 | 533 | 0.65 | 0.73 |
| 1000 | 50000 | 1401 | 1401 | 158 | 163 | 6.47 | 6.49 |
| 1000 | 500000 | 14013 | 14016 | 500 | 611 | 2.04 | 2.03 |

**Table 8.** Evenly spaced transmitters, variable power

| M | N | Amplitude | | Ampl. Variation | | Phase Variation | |
|---|---|---|---|---|---|---|---|
| | | Theory | Exp | Theory | Exp | Theory | Exp |
| 10 | 500 | 158.1 | 175.9 | 15 | 38 | 5.44 | 10.06 |
| 10 | 5000 | 1581 | 1543 | 47 | 82 | 1.71 | 0.56 |
| 10 | 50000 | 15811 | 15770 | 150 | 87 | 0.54 | 0.13 |
| 10 | 500000 | 158114 | 158088 | 474 | 52 | 0.17 | 0.01 |
| 100 | 5000 | 500 | 511 | 49 | 65 | 5.70 | 5.91 |
| 100 | 50000 | 5000 | 5072 | 157 | 506 | 1.80 | 0.44 |
| 100 | 500000 | 50000 | 50101 | 497 | 717 | 0.57 | 0.04 |
| 1000 | 50000 | 1581 | 1578 | 158 | 149 | 5.72 | 4.32 |
| 1000 | 500000 | 15811 | 15789 | 500 | 490 | 1.81 | 0.50 |

**Table 9.** Evenly spaced transmitters of constant power

| M | N | Amplitude | | Ampl. Variation | | Phase Variation | |
|---|---|---|---|---|---|---|---|
| | | Theory | Exp | Theory | Exp | Theory | Exp |
| 10 | 500 | 140.1 | 149.9 | 15.8 | 33.8 | 6.46 | 10.2 |
| 10 | 5000 | 1401 | 1394 | 50 | 83 | 2.04 | 1.52 |
| 10 | 50000 | 14013 | 14100 | 158 | 171 | 0.65 | 0.28 |
| 10 | 500000 | 140125 | 140978 | 500 | 321 | 0.20 | 0.06 |
| 100 | 5000 | 443 | 448 | 50 | 66 | 6.47 | 6.47 |
| 100 | 50000 | 4431 | 4469 | 158 | 463 | 2.04 | 0.97 |
| 100 | 500000 | 44311 | 44377 | 500 | 673 | 0.65 | 0.30 |
| 1000 | 50000 | 1401 | 1400 | 158 | 149 | 6.47 | 5.26 |
| 1000 | 500000 | 14013 | 14001 | 500 | 489 | 2.04 | 1.10 |

in the case of variable power, and Table 9 shows the results of the same experiment using constant power.

Except in the smallest cases, the variance in the amplitude seems to be a little more than predicted by the random model, and the variance in phase somewhat less. The random model still provides a fairly good prediction, however.

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